JOURNAL OF COMBINATORIAL THEORY, Series B 58, 280–290 (1993)

On the Nash-Williams' Lemma in Graph Reconstruction Theory

B. D. THATTE

Department of Mathematics, Indian Institute of Science, Bangalore 560012, India

Received September 20, 1990

A generalization of Nash-Williams' lemma is proved for the structure of m-uniform null $(m-k)$-designs. It is then applied to various graph reconstruction problems. A short combinatorial proof of the edge reconstructibility of digraphs having regular underlying undirected graphs (e.g., tournaments) is given. A type of Nash-Williams' lemma is conjectured for the vertex reconstruction problem. © 1993 Academic Press, Inc.

INTRODUCTION

Let X be an N element set. By 2^X and $X^{(m)}$ we denote, respectively, the power set of X and the family of all the subsets of cardinality m. We define an $\binom{N}{m} \times \binom{N}{m-k}$ matrix H^k_m as follows: The rows of H^k_m are indexed by the sets A_i from the family $X^{(m)}$ and the columns of H^k_m are indexed by the sets B_j from the family $X^{(m-k)}$. An element h^k_{ij} of this matrix is 1 if $B_j \subset A_i$ and 0 otherwise. Let $x = (x_i;\ i = 1 \text{ to } \binom{N}{m})$ be a row vector, whose entries are indexed by $A_i \in X^{(m)}$. As pointed out in [GKR], a vector x such that $xH^k_m = 0$ is the same as an m-uniform null $(m-k)$-design.

The main result of Section I is the result on the structure of x when $xH^k_m = 0$ (Theorem 1.1). In Section II we give several applications of Theorem 1.1 to the edge reconstruction, reconstruction from the shuffled edge deck, reconstruction from the k-edge deck etc. We demonstrate that several earlier results (results of Nash-Williams [N], Lovász [L] and Müller [M] and Alon *et al.* [ACKR]) follow from Theorem 1.1 in more general forms. In Section III we prove, using a simple combinatorial argument, the edge reconstructibility of directed graphs having regular underlying undirected graphs. This result, as a special case, proves a result of Harary and Palmer [HP] that tournaments are edge reconstructible. In the last section we conjecture an analogue of Nash-Williams' lemma for the vertex reconstruction conjecture. We also give some examples supporting the conjecture.

0095-8956/93 $5.00

I. THE MAIN THEOREM ON THE STRUCTURE OF x

THEOREM 1.1. *If $xH_m^k = 0$ then for arbitrary fixed sets B and A_l such that $B \subseteq A_l$ and $A_l \in X^{(m)}$ we have*

$$\sum_{i \mid A_i \cap B = T} x_i = \sum_{\substack{Y \mid T \subseteq Y \subseteq B \\ |Y| \geqslant m-k+1}} (-1)^{|Y|-|T|} \sum_{i \mid A_i \supseteq Y} x_i$$

for every T such that $T \subseteq B$.

To prove this we first prove the following two lemmas.

LEMMA 1.2. *If $xH_m^k = 0$ then $xH_m^{k'} = 0 \ \forall k' \geqslant k$.*

Proof. Choose any set $C \subset X$ such that $|C| = m-k-1$. Consider all the $(m-k)$-sets containing C. Each m-set that contains C, contains exactly $k+1$ subsets containing C and having cardinality $m-k$. Thus

$$\sum_{i \mid A_i \supseteq C} x_i = \frac{1}{k+1} \sum_{\substack{B \mid B \supseteq C, \\ |B| = m-k}} \sum_{i \mid A_i \supseteq B} x_i.$$

But the inner sum on the RHS is 0 because $xH_m^k = 0$. Therefore, $\sum_{i \mid A_i \supseteq C} x_i = 0$. As C is arbitrary, we have $xH_m^{k+1} = 0$. This implies the lemma.

The next lemma is the Möbius inversion technique.

LEMMA 1.3. *Let a and b be two real valued functions defined on 2^X. For $S \in 2^X$, let $a: S \to a_S$ and $b: S \to b_S$. If the two functions are such that $a_S = \sum_{Y \supseteq S} b_Y$ then we can write $b_S = \sum_{Y \supseteq S} (-1)^{|Y|-|S|} a_Y$.*

The proof of this is straightforward and we omit it.

Proof of Theorem 1.1. As stated in the statement of Theorem 1.1 we fix $S \subseteq B \subseteq A_l$, $A_l \in X^{(m)}$. For any $T \in 2^X$, we define

$$\begin{aligned} b_T &= \sum_{i \mid A_i \cap B = T} x_i \qquad \text{when} \quad T \supseteq S \\ &= 0 \qquad\qquad\qquad \text{otherwise,} \end{aligned}$$

and

$$a_T = \sum_{i \mid A_i \cap B \supseteq T \cup S} x_i.$$

Now

$$\sum_{Y \supseteq T} b_Y = \sum_{Y \supseteq T \cup S} \sum_{i \mid A_i \cap B = Y} x_i = \sum_{i \mid A_i \cap B \supseteq T \cup S} x_i = a_T.$$

Thus the functions defined above, satisfy the conditions of Lemma 1.3 and we have, when $T \supseteq S$,

$$\sum_{i \mid A_i \cap B = T} x_i = \sum_{Y \supseteq T} (-1)^{|Y| - |T|} \sum_{i \mid A_i \cap B \supseteq Y \cup S} x_i.$$

When $S \subseteq T \subseteq B$ and $xH^k_m = 0$, we obtain

$$\sum_{i \mid A_i \cap B = T} x_i = \sum_{\substack{Y \mid T \subseteq Y \subseteq B, \\ |Y| \geqslant m - k + 1}} (-1)^{|Y| - |T|} \sum_{i \mid A_i \cap B \supseteq Y} x_i. \tag{1.1}$$

This completes the proof of Theorem 1.1.

Remark. Bondy observed (see Section 10 in [B]) that this is just a different formulation of the Frankl–Pach lemma in design theory (whose proof may be found in [B]).

II. Applications of Theorem 1.1

Let $\Gamma_g = \{G_1, ..., G_r\}$ and $\Gamma_h = \{H_1, ..., H_r\}$ be two sets of m-edge spanning subgraphs of a labelled complete graph K_n, such that $[G_i] \neq [H_j]\ \forall 1 \leqslant i, j \leqslant r$. (Here by $[G]$ we denote the isomorphism class of a labelled graph G.) Let S^i_g denote the k-edge deck of G_i, i.e., the collection of all the unlabelled k-edge deleted subgraphs of G_i. By S_g we will denote the shuffled k-edge deck (k-SED) of Γ_g, i.e., $\bigcup_{i=1}^r S^i_g$ (where the union is the "multiset union"). We call it just the shuffled edge deck (SED) when $k = 1$. In [T1], the author studied the problem of reconstructing Γ_g up to isomorphism from its SED (i.e., the problem of proving that Γ_g and Γ_h are same up to isomorphism classes of the graphs in these sets when $S_g = S_h$). This problem is relevant to the vertex reconstruction problem and also is interesting in its own right.

To apply Theorem 1.1 to reconstruction problems, we need some notation and also have to define the matrix H^k_m and the vector x in a particular manner. We will denote the automorphism group of a graph by aut G. We consider a fixed labelled complete graph K_n and imagine all the graphs under consideration as fixed spanning subgraphs of K_n. For graphs G and H and a spanning subgraph B of G, we define $(H, G, B) = |\{f \in \text{aut } K_n \text{ s.t. } f(H) \cap G = B\}|$, where $f(H)$ and $f(H) \cap G$ have the usual meanings. To define the matrix H^k_m, we choose the set X as the edge set of K_n; then the elements of $X^{(m)}$ denote the m-edge graphs with vertex set $V(K_n)$ and the elements of $X^{(m-k)}$ denote the $(m-k)$-edge graphs on the same vertex set. The m-edge and $(m-k)$-edge graphs are respectively denoted by A_i and B_j. Define a row vector x_G whose entries are indexed by $A_i \in X^{(m)}$, as follows. The ith entry

$$\begin{aligned} (x_G)_i &= 1 \qquad \text{if} \quad [A_i] = [G] \\ &= 0 \qquad \text{otherwise.} \end{aligned}$$

For $G_i \in \Gamma_g$ and $H_i \in \Gamma_h$, define the vector x by

$$x = \sum_{i=1}^{r} \{x_{G_i} |\text{aut } G_i| - x_{H_i} |\text{aut } H_i|\}.$$

We assume, in all the applications below, that x and H_m^k are as defined above. The applications of Theorem 1.1 to reconstruction are based on the following lemma.

Lemma 2.1. *If Γ_g, Γ_h are two nonisomorphic reconstructions of the same k-SED (i.e., $S_g = S_h$ but $[G_i] \neq [H_j]\ \forall 1 \leqslant i, j \leqslant r$) and x and H_m^k are as defined above then we have $xH_m^k = 0$.*

Proof of this is on the same lines of that in [GKR] except that we use an analogue of Kelly's lemma (see [K]) for the shuffled k-edge deck rather than k-edge deck. An analogue of Kelly's lemma for k-SED can be proved in the same manner as Kelly's lemma. We skip the details here.

In applications (A1) to (A4) we assume $k = 1$, $S = \phi$, $B = A_l$ and $T \subseteq A_l$.

(A1) With these assumptions, $\sum_{i \mid A_i \supseteq Y} x_i = 0\ \forall Y$ s.t. $|Y| \leqslant m-1$ and Eq. (1.1) gives

$$\sum_{i\,:\,A_i \cap A_l = T} x_i = (-1)^{m-|T|} x_l. \tag{2.2}$$

This is exactly analogous to Nash-Williams' lemma. When Γ_g, Γ_h are two nonisomorphic reconstructions of the same SED then Lemma 2.1 along with Eq. (2.2) gives

$$\sum_{i=1}^{r} (G_i, G_l, T) - \sum_{i=1}^{r} (H_i, G_l, T) = (-1)^{m-|E(T)|} |\text{aut } G_l|\, \lambda \tag{2.3}$$

for any fixed G_l in Γ_g and for any spanning subgraph T of G_l, where λ is the number of graphs in Γ_g isomorphic to G_l. For $r = 1$, this is the same as the Nash-Williams' lemma for the edge reconstruction.

(A2) Lovász's result [L] (that $m > \frac{1}{2}\binom{n}{2}$ implies the edge reconstructibility of an m-edge, n-vertex graph) remains unchanged even for the shuffled edge deck problem. This is because if we take $T = \phi$ and fix A_l such that x_l is nonzero then from Eq. (2.2), we obtain A_i such that $A_i \cap A_l = \phi$.

Thus if X does not have $2m$ distinct elements then $xH^1_m=0$ implies that $x=0$. This, in case of the shuffled edge deck problem says that if $m>\frac{1}{2}\binom{n}{2}$ then Γ_g is reconstructible up to the isomorphism classes of the graphs in it, from its SED.

(A3) Müller's result [M] changes by an additional $\log_2 r$ term, i.e., $m>n\log_2 n-n+\log_2 r$ implies that we can obtain from the SED, the set Γ_g up to the isomorphism classes of the graphs in it. This can be proved from Eq. (2.3) as follows: when we fix $G_l\in\Gamma_g$ and consider 2^m different spanning subgraphs of G_l in Eq. (2.3), we obtain at least 2^m nonzero entries in x. In the isomorphism class of a graph G, there are exactly $n!/|\mathrm{aut}\, G|$ graphs, so x has at most $2rn!$ nonzero entries. Thus when $2^{m-1}>rn!$, we have the reconstructibility from the SED. This gives an additional $\log_2 r$ term in Müller's result.

Thus Lovász's result is more general than Müller's result in the sense that it gives the same lower bound on the number of edges, irrespective of r.

(A4) In the problem of reconstruction of a set of claw free, chordal of P_4 free graphs from the shuffled edge deck, the elemination of K_4 can be done in a similar way as in [EPY]. This is demonstrated in [T1], where the problem of edge reconstruction of set of connected graphs is solved for claw free and P_4 free and a large class of chordal graphs.

In the following three applications, we assume that $k>1$.

(A5) To apply Eq. (1.1) to k-edge reconstruction, we fix sets S and A_l such that x_l is nonzero, $S\subseteq A_l$, and $|S|\leqslant m-k$. We claim that there must exist a set B such that $|B|\geqslant m-k+1$ and $\sum_{i\mid A_i\cap B=T}x_i$ is nonzero for all T such that $S\subseteq T\subseteq B$. This follows from the fact that x_l is nonzero and that for all Y such that $|Y|\leqslant m-k$ we have $\sum_{i\mid A_i\supseteq Y}x_i=0$. Thus $\exists Y_1$ such that $S\subseteq Y_1\subseteq A_l$, $|Y_1|\geqslant m-k+1$, and $\sum_{i\mid A_i\supseteq Y_1}x_i$ is nonzero and $\sum_{i\mid A_i\supseteq Y}x_i=0$ for all Y such that $S\subseteq Y\subset Y_1$. We choose $B=Y_1$. Thus $\forall T$ such that $S\subseteq T\subseteq B$, we have

$$\sum_{i\mid A_i\cap B=T}=(-1)^{|B|-|T|}\sum_{i\mid A_i\supseteq B}x_i\neq 0. \tag{2.3}$$

This is exactly analogous to the generalization of Nash-Williams' lemma for k-edge reconstruction, which was proved in [ACKR]. But Eq. (2.3) can be used also for reconstruction from k-SED. Thus for any fixed $G_l\in\Gamma_g$ and a spanning subgraph S of G_l such that $|E(S)|\leqslant m-k$, $\exists$ a spanning subgraph B of G_l with $|E(B)|\geqslant m-k+1$, such that $\forall$ spanning subgraphs T of G_l with $E(S)\subseteq E(T)\subseteq E(B)$, we have

$$\sum_{i=1}^{r}(G_i,B,T)-\sum_{i=1}^{r}(H_i,B,T)=(-1)^{|E(B)|-|E(T)|}\sum_{i\mid E(A_i)\supseteq E(B)}x_i\neq 0. \tag{2.4}$$

(A6) The analogue of Lovász's result is the same for reconstruction from the shuffled k-edge deck as from the k-edge deck; i.e., if $2(m-k+1)+k-1>\binom{n}{2}$ (or $2m \geqslant \binom{n}{2}+k$) then Γ_g is reconstructible up to the isomorphism classes of the graphs in it, from its k-SED.

(A7) Te result of [GKR] (that $m-k>n\log_2 n-n$ implies k-edge reconstructibility) improves by an additional $\log_2 r$ term in case of reconstruction from k-SED.

Claims in (A6) and (A7) follow from (2.4) if we consider S to be a graph with empty edge set.

(A8) In (1.1), let $S=\phi$ and $B=A_l$, where A_l is such that x_l is nonzero. If for every Y such that $|Y| \geqslant m-k+1$, all the nonzero x_i such that $A_i \supseteq Y$ have the same sign then the RHS of Eq. (1.1) is nonzero. This is because each nonzero x_i for which $A_i \supseteq T$ and $|A_i \cap A_l| = z$, appears exactly $\sum_{|Y|=m-k+1}^{z} \binom{z-|T|}{|Y|-|T|}(-1)^{|Y|-|T|}$ times on the RHS of Eq. (1.1). It can be proved that

$$(-1)^{m-k+1-|T|} \sum_{|Y|=m-k+1}^{z} \binom{z-|T|}{|Y|-|T|}(-1)^{|Y|-|T|} > 0 \quad \text{when} \quad |T| \leqslant m-k. \tag{2.5}$$

The proof of this is given as an appendix at the end. In the case of k-edge reconstruction, this means the following: when G and H are two nonisomorphic reconstructions from the same k-edge deck such that no $m-k+1$ edge graph is isomorphic to a subgraph of both G and H then $(-1)^{m-k+1-|E(T)|}[(G,G,T)-(H,G,T)]$ is positive for all spanning subgraphs T of G such that $|E(T)| \leqslant m-k$. For $k=1$, this is the usual Nash-Williams' lemma. The above result further implies that if $[\binom{m}{m-k}+\binom{m}{m-k-2}+\cdots+\binom{m}{0 \text{ or } 1}] > n!/|\text{aut } H|$ then at least one $(m-k+1)$-edge subgraph of G is isomorphic to a subgraph of H. An analogue of this is also valid for reconstruction from k-SED. Thus for reconstructing only "partially" (i.e., for reconstructing only some $(m-k+1)$-edge subgraph), we obtain a better lower bound than $n\log_2 n-n+k$ on the number of edges.

Some general results about Nash-Williams' lemma and some of its recent applications may be found in [ACKR; KR1; KR2].

III. A Combinatorial Technique for Edge Reconstruction

Theorem 3.1. *Directed graphs having regular underlying undirected graphs are edge reconstructible.*

Proof. Recognition. This is trivial because the underlying undirected graph is regular and, therefore, trivially edge reconstructible.

Reconstruction. Suppose that the graph is not edge reconstructible. Let G and H be two labelled nonisomorphic directed graphs having a regular underlying undirected graph and having the same edge deck. Let F be the regular underlying undirected graph of G. The underlying undirected graph of H is isomorphic to F. Now we can identify uniquely the pair of vertices in an edge deleted subgraph of G, where we must put a directed edge. Thus for each directed edge (a, b) in G we obtain a reconstruction $H_1 = G-(a,b)+(b,a)$, which is nonisomorphic to G. (Obviously H_1 is isomorphic to H: as only (b, a) can be the replacing edge of (a, b), there are at most two nonisomorphic reconstructions possible.) Similarly if we change the direction of any edge of H_1, we obtain a reconstruction isomorphic to G. Thus, from a fixed underlying undirected graph, depending upon the choice of the direction for each edge, we obtain 2^{m-1} reconstructions isomorphic to G and 2^{m-1} reconstructions isomorphic to H. There are exactly $n!/|\text{aut } F|$ graphs on the same vertex set, isomorphic to F, and each one of them gives 2^{m-1} distinct graphs isomorphic to G. The number of graphs isomorphic to G is $n!/|\text{aut } G|$. Thus,

$$\frac{n!}{|\text{aut } G|} = \frac{n!}{|\text{aut } F|} 2^{m-1}$$

$$\Rightarrow 2^{m-1} = \frac{|\text{aut } F|}{|\text{aut } G|}$$

$$\Rightarrow 2^{m-1} \mid |\text{aut } F|$$

$$\Rightarrow 2^{m-1} \mid n! \qquad \text{as} \quad |\text{aut } F| \mid n!$$

Now, the number of powers of 2 in $n!$ is $\lfloor n/2 \rfloor + \lfloor n/4 \rfloor + \cdots + 1$ which is less than $n(\frac{1}{2} + \frac{1}{4} + \cdots) = n$. Therefore, $m \leqslant n$.

If the graph is disconnected and is a disjoint union of directed graphs whose underlying undirected graphs are cycles then it is trivially edge reconstructible. Thus we assume that F itself is a cycle. In this case, if the graph is not edge reconstructible then by changing the directions of some of the edges, we can prove that either G or H (say G) is a directed cycle. As the graphs obtained by changing the directions of the even number of edges of G should be isomorphic to G, we change the directions of just two edges of the directed cycle. The resulting graph is not a directed cycle (i.e., not isomorphic to G), which is a contradiction. This completes the proof.

Remarks. (i) This theorem, as a special case, proves that tournaments are edge reconstructible, which was proved by Harary and Palmer [HP].

(ii) The same technique, in fact, proves the edge reconstructibility of digraphs whose underlying undirected graphs have no two vertices differing in their degrees by 1.

(iii) By a similar method we can prove the following result for undirected graphs. Let G and H be two graphs having the same edge deck such that every edge e of G has exactly one replacing edge f such that $[G-e+f]=[H]$ and every edge set $(e_i, e_j) \subseteq E(G)$ has exactly one replacing edge set (f_i, f_j) with the property that $[G-(e_i, e_j)+(f_i, f_j)]=[G]$, then G is isomorphic to H. There may not exist graphs satisfying the conditions of this result but it will be interesting to identify classes of graphs for which the assumption of nonreconstructibility will imply the conditions of this result. We ask the question: can we prove the edge reconstructibility of the class of maximal planar graphs (for which the vertex reconstructibility is also known [FL]) by this method?

IV. Nash-Williams' Lemma and Vertex Reconstruction

Let G and H be fixed spanning subgraphs of a labelled complete graph K_n and let A be any subset of the vertex set of K_n. Let $G|_A$ and $H|_A$ be the subgraphs of G and H, respectively, induced by the vertex set A. We say that A is a maximal intersection of G and H if $G|_A = H|_A$ and there is no $x \in V(K_n) - A$ such that $G|_{A \cup \{x\}} = H|_{A \cup \{x\}}$. There may exist many maximal sets of intersection of G and H.

Suppose that there is a pair of nonisomorphic graphs G and H which is a counterexample to the edge reconstruction conjecture (ERC). Then Nash-Williams' lemma says that for every edge set $E \subseteq E(G)$, there exists an edge set F such that $F \cap E(G) = \phi$ and $[G-E+F]=[G]$ or $[H]$, depending respectively on whether $|E|$ is even or odd. We expect an exact analogue of this for the counterexamples to the vertex reconstruction conjecture (VRC) (if there exist any).

Hypothetical Nash-Williams' Lemma. *Conjecture. Let G and H be two nonisomorphic spanning subgraphs of a labelled complete graph K_n, having the same collection of unlabelled vertex deleted subgraphs. If $n-|A|$ is odd then there exists $f \in \operatorname{aut} K_n$ such that A is a maximal intersection of $f(H)$ and G and if $n-|A|$ is even then there exists $f \in \operatorname{aut} K_n$ such that A is a maximal intersection of $f(G)$ and G.*

If this is true then, for $n>2$, it will immediately imply the following: if all the vertex degrees are $>n/2$ then the graph is vertex reconstructible (to prove this take A as one vertex set). This will be an analogue of Lovász's result. To support this expectation we do not know any counterexamples to the vertex reconstruction conjecture, except the two vertex

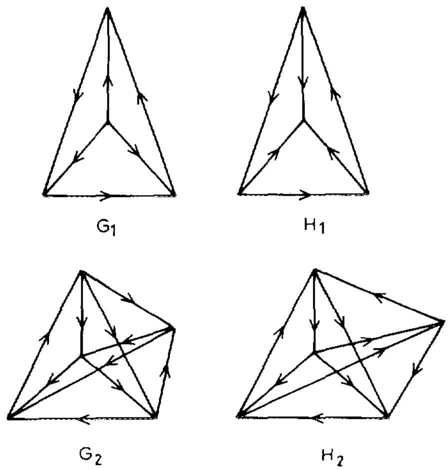

FIGURE 1

counterexamples $[G] = K_2$ and $[H] = K_1 \cup K_1$. This example trivially satisfies the hypothetical Nash-Williams' lemma. But we have many counterexamples for digraphs (see [S1; S2; S3]). It can be verified that the following counterexample tournament pairs (Fig. 1) satisfy the above conjecture. We conjecture that our hypothetical Nash-Williams' lemma is a characterization of counterexample pairs of tournaments.

In case of vertex reconstruction we know that a graph is vertex reconstructible iff its complement is vertex reconstructible. So if we have the above analogue of Lovász's result then that will mean that if all degrees are less than $n/2$ then the graph is vertex reconstructible. Thus we can expect a counterexample only in graphs in which some of the degrees are greater than $n/2$ and some are less than $n/2$. In this context it is interesting to observe that the above counterexamples for digraphs contain some vertices with indegree $<n/2$ and outdegree $>n/2$ and some vertices with indegree $>n/2$ and outdegree $<n/2$.

APPENDIX: PROOF OF EQ. (2.5)

We change the variables and prove that

$$P = (-1)^a \sum_{t=a}^{x} (-1)^t \binom{x}{t} > 0 \qquad \text{when} \quad a > 0.$$

Let $x = a + c$ and $t = a + b$; therefore, we should prove that

$$P = \sum_{b=0}^{c} (-1)^b \binom{a+c}{a+b} > 0.$$

This can be done by induction on c. When $c=0$, $\sum_{b=0}^{c}(-1)^b\binom{a+c}{a+b}=1$ for all $a>0$.

Now let $\sum_{b=0}^{c}(-1)^b\binom{a+c}{a+b}$ be positive for all $0\leqslant c\leqslant p$ and for all $a>0$. When $c=p+1$.

$$P=\sum_{b=0}^{p+1}(-1)^b\binom{a+p+1}{a+b}=\sum_{b=0}^{p}(-1)^b\binom{a+p+1}{a+b}+(-1)^{p+1}$$
$$=\sum_{b=0}^{p}(-1)^b\binom{a+p}{a+b}+\sum_{b=0}^{p+1}(-1)^b\binom{a-1+p+1}{a-1+b}.$$

By expanding the second term further we obtain

$$P=\sum_{b=0}^{p}(-1)^b\binom{a+p}{a+b}+\sum_{b=0}^{p}(-1)^b\binom{a-1+p+1}{a-1+b}+\cdots$$
$$+\sum_{b=0}^{p}(-1)^b\binom{2+p}{2+b}+\sum_{b=0}^{p}(-1)^b\binom{1+p}{1+b}.$$

By the induction hypothesis, each term of the above expression is positive. This proves the desired result.

Acknowledgments

The work presented here is based on my Ph.D. thesis [T2] and I thank my advisor Dr. N. Manickam for his constant encouragement throughout the work. I am also thankful to the referees for many useful comments; in particular I thank one of the referees for suggesting to me the use of Möbius inversion, which simplified the proof of Theorem 1.1.

References

[ACKR] N. Alon, Y. Caro, I. Krasikov, and Y. Roditty, Combinatorial reconstruction problems, *J. Combin. Theory Ser. B* **47** (1989), 153–161.

[B] J. A. Bondy, A graph reconstructor's manual, preprint, 1991.

[EPY] M. N. Ellingham, L. Pyber, and X. Yu, Claw free graphs are edge reconstructible, preprint, 1989.

[FL] S. Fiorini and J. Lauri, The reconstruction of maximal planar graphs, *J. Combin. Theory Ser. B* **30** (1981), 188–195.

[GKR] C. Godsil, I. Krasikov, and Y. Roditty, A note on k-edge reconstruction problem, *J. Combin. Theory Ser. B* **43**, No. 3 (1987), 360–363.

[HP] F. Harary and E. Palmer, On the problem of constructing a tournament from subtournaments, *Monatsh. Math.* **1** (1987), 14–23.

[K] P. J. Kelly, A congruence theorem for trees, *Pacific J. Math.* **7** (1957), 961–968.

[KR1] I. Krasikov and Y. Roditty, Some applications of the Nash-Williams' lemma to the edge reconstruction conjecture, *Graphs Combin.* **6** (1990), 37–39.

[KR2] I. Krasikov and Y. Roditty, Recent applications of the Nash-Williams' lemma to the edge reconstruction conjecture, *Ars Combin.* **29A** (1990), 215–224.

[L] L. Lovász, A note on the line reconstruction problem, *J. Combin. Theory Ser. B* **13** (1972), 309–310.

[M] V. Müller, The edge reconstruction hypothesis is true for graphs with more than $n \log_2 n$ edges, *J. Combin. Theory Ser. B* **22** (1977), 281–283.

[N] C. St. J. A. Nash-Williams, The reconstruction problem, *in* "Selected Topics in Graph Theory" (L. Beineke and R. Wilson, Eds.), Chap. 8, Academic Press, London, 1978.

[S1] P. Stockmeyer, The falsity of the reconstruction conjecture for tournaments, *J. Graph Theory* **1** (1977), 19–25.

[S2] P. Stockmeyer, A census of non-reconstructible digraphs. I. Six related families, *J. Combin. Theory Ser. B* **31** (1981), 232–239.

[S3] P. Stockmeyer, A census of non-reconstructible digraphs. II. A family of tournaments, preprint.

[T1] B. D. Thatte, Some results and approaches for reconstruction conjectures, *in* "First Malta Conference on Graphs and Combinatorics, May–June 1990."

[T2] B. D. Thatte, "On the Reconstruction Problems in Graph Theory," Ph.D. thesis, Indian Institute of Science, Bangalore, India, March 1990.